A Generalized Chebyshev Suspended Substrate Stripline Bandpass Filter

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Abstract —**A design method for narrow-band suspended substrate stripline filters having a true bandpass structure is presented. A generalized Chebyshev low-pass prototype is used, resulting in a convenient form of realization in suspended substrate stripline.**

A prototype device, designed with the aid of a computer program, is given as an example. Results from this device show that the method of realizing such a filter is viable for many applications and may be suitable to replace more conventional types of microwave filter realized using TEM-mode resonators.

I. Introduction

EXISTING DESIGNS of suspended substrate stripline filters are available [1] to produce discrete low-pass and high-pass devices which have high selectivity and low insertion loss. Bandpass filters may therefore be constructed by cascading low-pass and high-pass sections. This technique is suitable for devices having wide bandwidths, but problems occur when attempting to realize fractional bandwidths of less than about 10 percent. This is due to the high degree needed for the filters to achieve good selectivity, and problems associated with roll-off at the band edges of the two discrete sections, causing large insertion losses and poor amplitude flatness.

Considering these factors, it would seem necessary to realize a true bandpass structure to achieve good narrowband performance. The use of suspended substrate techniques will allow highly selective prototypes to be used which can achieve superior performance to designs derived from the maximally flat or conventional Chebyshev prototypes. The exact elliptic function prototype will realize a given selectivity and passband ripple level with the minimum degree possible. The impedance variation produced, however, when attempting to realize a prototype of this kind, is large and hence presents problems for printed circuit realization.

The odd degree generalized Chebyshev prototype [2] with one transmission zero at infinity has a selectivity nearly as good as the same degree elliptic function prototype and has a much smaller impedance change through the network. This generalized Chebyshev prototype (Fig. 1) was therefore used in the design.

The frequency response of this prototype is shown in Fig. 2 and has $n-1$ transmission zeros at a frequency ω_0 close to bandedge, where n is the degree of the low-pass prototype.

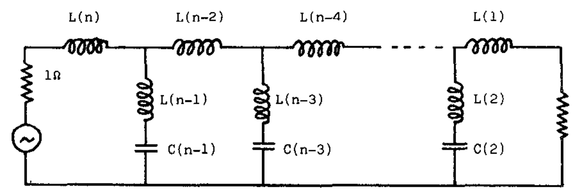

Fig. 1. Generalized Chebyshev low-pass prototype.

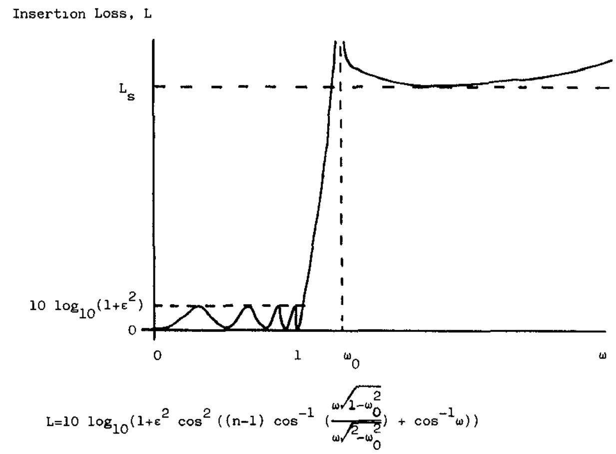

Fig. 2. Insertion loss characteristic of generalized Chebyshev low-pass prototype.

II. Lumped Circuit Development

To produce the required structure, the bandpass transformation of (1) is applied to all the elements of the prototype of Fig. 1

$$p' = \frac{\omega_c}{B}\left[\frac{P}{\omega_c} + \frac{\omega_c}{p}\right] \tag{1}$$

where $B = \omega_2 - \omega_1$, $\omega_c = \sqrt{\omega_1\omega_2}$, and $p = j\omega$ for sinusoidal input. ω_1 and ω_2 are the bandedge frequencies of the bandpass network.

This results in the network of Fig. 3, which has the frequency response indicated in Fig. 4. Examining this network, we see there are n complex shunt networks of the form of Fig. 5(a). Since these are very difficult to realize directly in suspended substrate stripline, a more convenient equivalent circuit must be found. Such an equivalent structure has been derived [3] and is shown in Fig. 5(b). We may

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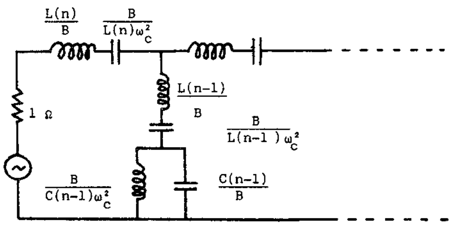

Fig. 3. Bandpass network.

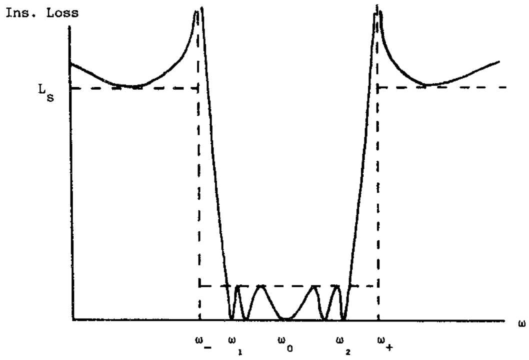

Fig. 4. Response of generalized Chebyshev bandpass filter.

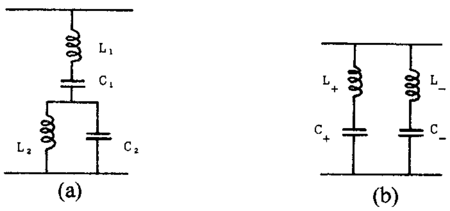

Fig. 5. Alternative configuration for shunt elements.

obtain the values of L_+, L_-, C_+, C_- by synthesizing the section as follows. For Fig. 5(a), we have

$$Y_a = \frac{C_1 p\left[L_2 C_2 p^2 + 1\right]}{1 + ap^2 + bp^4} \tag{2}$$

where

$$a = L_1C_1 + L_2C_2 + L_2C_1$$

and

$$b = L_1L_2C_1C_2.$$

Splitting (2) into partial fractions gives the result

$$Y_a = \frac{\beta_+ p}{1 + \alpha_+ p^2} + \frac{\beta_- p}{1 + \alpha_- p^2} \tag{3}$$

where

$$\alpha_{+/-} = \frac{a}{2} \pm \sqrt{(a/2)^2 - b}$$

$$\beta_+ = \frac{C_1L_2C_2 - C_1\alpha_+}{\alpha_- - \alpha_+}$$

$$\beta_- = \frac{C_1L_2C_2 - C_1\alpha_-}{\alpha_+ - \alpha_-}.$$

Considering the admittance of the network of Fig. 5(b) we can immediately write

$$Y_b = \frac{C_+ p}{L_+C_+ p^2 + 1} + \frac{C_- p}{L_-C_- p^2 + 1}. \tag{4}$$

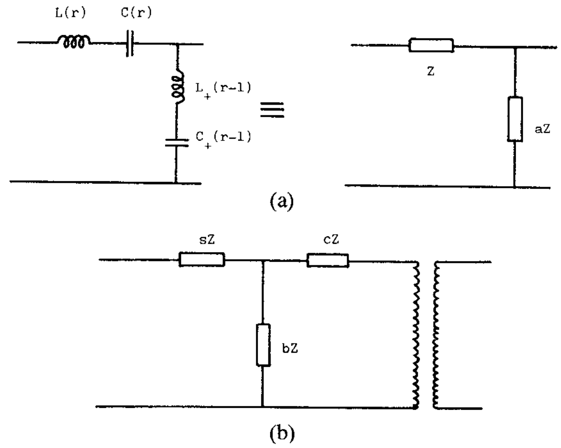

Fig. 6. Introduction of redundant series elements.

Equating Y_a and Y_b gives the identities

$$C_+ = \beta_+ \qquad C_- = \beta_-$$

and

$$L_+ = \frac{\alpha_+}{\beta_+} \qquad L_- = \frac{\alpha_-}{\beta_-}. \tag{5}$$

We now have pairs of series resonant shunt circuits which resonate at frequencies ω_+ and ω_- located at either side of the passband. These resonant sections, however, are not physically separated, thus making realization in printed form very difficult. What is required is an extra element or elements between these two sections which will allow separation. A series LC circuit, resonant in the center of the passband, may be introduced in a redundant manner into the network to provide the separation as follows.

This extra section may be derived by considering the networks of Fig. 6. The two-port impedance matrices for Fig. 6(a) and (b), respectively, may be written as

$$[Z_1] = \begin{bmatrix} Z(a+1) & aZ \\ aZ & aZ \end{bmatrix}$$

$$[Z_2] = \begin{bmatrix} Z(b+d) & mbZ \\ mbZ & m^2Z(b+c) \end{bmatrix}. \tag{6}$$

Equating the terms of these two matrices gives the identities

$$b = \frac{a}{m} \quad c = a\left[\frac{1}{m^2} - \frac{1}{m}\right] \quad d = a\left[1 - \frac{1}{m}\right] + 1. \tag{7}$$

Applying a similar procedure to the adjacent series resonant circuit and cascading the two networks results in the $1:m$ transformers cancelling to give the network of Fig. 7. The overall result is that of introducing the required extra LC circuit between the shunt resonators. The new element values are derived from the identities of (7). For a low-pass prototype of degree n the above process is repeated $(n-1)/2$ times for each of the pairs of shunt resonant sections in the circuit. The overall result is a network of the form of Fig. 8. To produce realizable element values, the transformer ratio m must be positive and

$$\frac{1}{m^2} - \frac{1}{m} > 0, \quad \text{hence } 0 < m < 1. \tag{8}$$

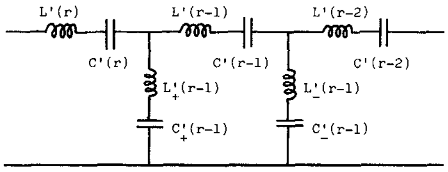

Fig. 7. Combination of equivalent networks.

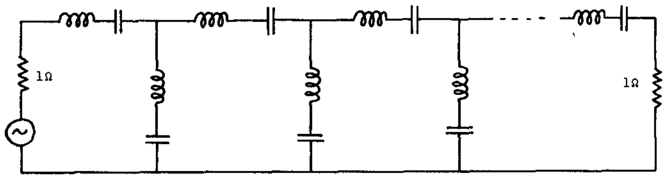

Fig. 8. Form of the final lumped bandpass network.

The actual value chosen for m is arbitrary but it is logical to make the impedance of the new series sections a similar value to that of the adjacent series sections at midband. This criteria, together with the fact that for narrow bandwidths, the resonant frequencies ω_+ and ω_- on either side of the passband are approximately equal, enables us to form the equation

$$2L_+(r-1)\cdot\left(\frac{1}{m^2}-\frac{1}{m}\right)=L(r)+L_+(r-1)\cdot\left(1-\frac{1}{m}\right). \tag{9}$$

This quadratic may then be solved to produce a value of m between 0 and 1.

We are now in a position to realize the circuit in suspended substrate stripline.

III. Device Realization

Considering first the shunt resonant elements, the impedance of a single section may be written as

$$Z=\frac{1-L_rC_r\omega^2}{jC_r\omega}. \tag{10}$$

This impedance is zero when

$$\omega=\omega_r=\frac{1}{\sqrt{L_rC_r}}. \tag{11}$$

To produce the required effect from a transmission line we consider the impedance of a length of line l terminated in an open circuit. We have

$$Z_{\text{in}}=\frac{Z_0}{j\tan\beta l} \tag{12}$$

where Z_0 is the characteristic impedance of the line and $\beta=\omega/cu_0$, cu_0 being the velocity of electromagnetic waves in free space as the dielectric is predominantly air. Equating this to (10) at a frequency ω_r gives the result $\tan(\beta l)=\infty$, hence

$$l=\frac{\pi u}{2\omega_r}. \tag{13}$$

To evaluate the characteristic impedance of the line, we equate the impedances of the LC network and that of the transmission line at the center of the passband ω_c. Hence

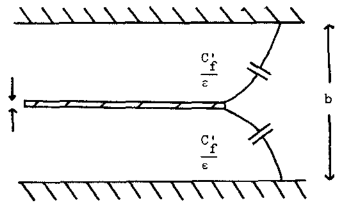

Fig. 9. Capacitive fringing off the ends of the shunt resonators.

$$\frac{Z_0}{j\tan\theta_c}=\frac{1-L_rC_r\omega_c^2}{jC_r\omega_c} \tag{14}$$

where

$$\theta_c=\frac{\pi\omega_c}{2\omega_r}.$$

Therefore

$$Z_0=\frac{1-(\omega_c/\omega_r)^2}{C_r\omega_c}\tan\left[\frac{\pi\omega_c}{2\omega_r}\right]. \tag{15}$$

Approximating (15) for narrow bandwidths ($\omega_c\simeq\omega_r$) gives

$$Z_0\simeq\frac{4}{C_r\pi\omega_c}. \tag{16}$$

This shows that for narrow bandwidths, the impedances of the shunt resonators are independent of bandwidth.

A further consideration in the design of the shunt resonators is the effect of fringing capacitance off the ends of the resonators which effectively increases their length. This is shown in Fig. 9. We may take this capacitance into account by equating it to an equivalent parallel plate capacitance and shortening the resonator by the amount indicated.

The values of fringing capacitance may be obtained [5], and with reference to Fig. 9 we may write

$$2l_f=\frac{C_f'}{\epsilon}(b-t) \tag{17}$$

where l_f is the effective extra length of the resonator and C_f'/ϵ is the fringing capacitance per unit width of the resonator.

Taking an example of a fifth-degree filter with a 10-percent bandwidth, the characteristic impedance of the resonators as derived from (15) is of the order of 5 Ω for 1-Ω filter terminations. If the impedance level were scaled to 50 Ω this would entail the realization of 250-Ω transmission lines, which is not practical in suspended substrate stripline. Therefore, the impedance level of the filter is scaled to a lower value (about 10-Ω terminations), and transformers are introduced at input and output to give the correct 50-Ω termination.

The transformers may be realized by means of a low-impedance transmission line one quarter wavelength long at bandcenter. The impedance required for the line is given by

$$Z_0=\sqrt{Z_{\text{in}}Z_L} \tag{18}$$

where $Z_{\text{in}}=50$ Ω and Z_L is the terminating impedance of the filter excluding the transformers.

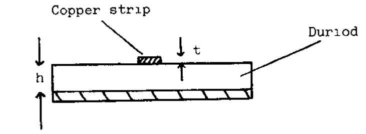

Fig. 10. Configuration of microstrip impedance transformers.

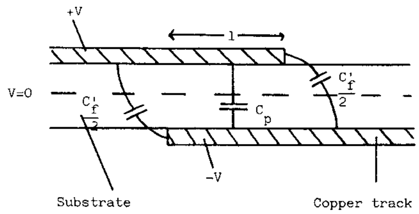

Fig. 11. Odd-mode excitation of inhomogeneous coupled lines.

As Z_L, and hence Z_0, is small, a microstrip configuration is used to realize the transformers as shown in Fig. 10. The substrate used for this design was RT/Duroid 5880, where h is 0.005 in. As the width of the strip is greater than h, we may use the identities [6]

$$Z_0 = \frac{\eta_0}{\sqrt{\epsilon_e}}\left[\frac{w}{h} + 1.393 + 0.667\ln\left(\frac{w}{h} + 1.444\right)\right]^{-1} \tag{19}$$

where

$$\epsilon_e = \frac{1}{2}\left[\epsilon_r + 1 + (\epsilon_r - 1)F\right]$$

and

$$F = \left(1 + \frac{12h}{w}\right)^{-1/2}.$$

ϵ_r is the relative dielectric constant of the Duroid substrate.

Hence, for a given width, the impedance of the line may be found, and as the impedance is roughly inversely proportional to strip width, an iterative procedure applied to (19) will converge rapidly to give the required width.

The series sections are required to couple across the ends of the shunt resonators and are realized by the configuration shown in Fig. 11. The overlapping lines not only possess capacitance but have a distributed inductance which has the effect of producing the series resonant circuit required [4].

As shown in Fig. 11, the circuit is operating in such a way as to make it necessary to consider only the odd-mode impedance when calculating the capacitance. If we consider the voltages on the upper and lower strips to be $+V$ and $-V$, then there is a plane in the center of the substrate for which $V = 0$. This enables us to consider a microstrip configuration where the ground plane spacing is $h/2$.

The static capacitance per unit length for this section may then be written in terms of the odd-mode microstrip impedance.

Hence

$$C_0 = \frac{\sqrt{\epsilon_e}}{uZ_{oo}} \tag{20}$$

where $u/\sqrt{\epsilon_e}$ is the odd-mode phase velocity.

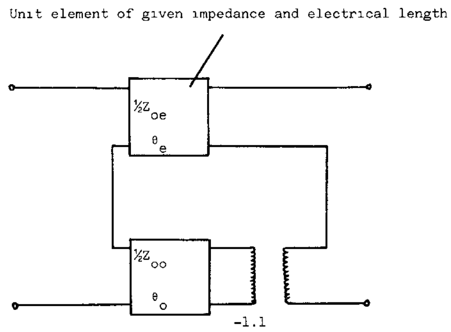

Fig. 12. Equivalent circuit of the configuration of Fig. 11.

The interstrip capacitance through the board will therefore be half the static capacitance per unit length, so

$$C_s = \frac{C_o l}{2} \tag{21}$$

where l is the length of the overlap.

Considering fringing effects off the sides and ends of the strips, it has been suggested [1] that for relatively small overlaps about 10 percent of the total capacitance results from fringing. Therefore, the length of overlap required to realize capacitance C_s is given by

$$l = \frac{1.8uZ_{oo}C_s}{\sqrt{\epsilon_e}}. \tag{22}$$

The value of Z_{oo} is given by replacing h in (19) by $h/2$.

We may now analyze the response of the overlap section to obtain its frequency of resonance. The equivalent circuit (4) is shown in Fig. 12, and the $ABCD$ parameters of the transfer matrix are

$$A = \frac{Z_{oe}\cot\theta_e + Z_{oo}\cot\theta_o}{Z_1} = D$$

$$B = \frac{j}{2}\,\frac{Z_{oe}^2 + Z_{oo}^2 - 2Z_{oo}Z_{oe}(\cot\theta_e\cot\theta_o + \csc\theta_e\csc\theta_o)}{Z_1}$$

$$C = \frac{2j}{Z_1} \tag{23}$$

where

$$Z_1 = Z_{oe}\csc\theta_e - Z_{oo}\csc\theta_o$$

and

$$\theta_e = \frac{\omega l}{u} \quad \theta_o = \frac{\omega l\sqrt{\epsilon_e}}{u}$$

as the dielectric is predominantly air for even-mode exitation.

To evaluate these $ABCD$ parameters, we must also find a value for the even-mode impedance of the section Z_{oe}. Now

$$Z_{oe} = \frac{\eta\epsilon\sqrt{\epsilon_r}}{C} \tag{24}$$

where η is the wave impedance of the medium and C is the total static capacitance of the strip. Hence, referring to Fig.

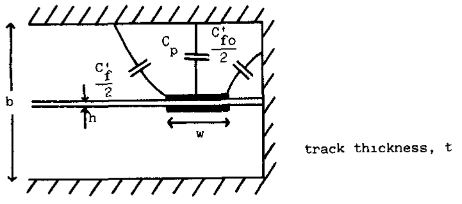

Fig. 13. Capacitance to ground of overlap section for even-mode excitation.

13, for an air dielectric we may write

$$Z_{oe}=\frac{\eta_0}{2(w/(b-h-2t))+C_f'+C_{fo}'}. \quad (25)$$

C_f' and C_{fo}' are the fringing capacitances off the corners and edges of the strip and may be determined from Getsingers graphs [5].

Using these *ABCD* parameters, the resonant frequency of the section may be determined, and it is found that to obtain sufficient inductance, the width of the strip must be impractically narrow.

If a realizable strip width is used, the remaining inductance may be realized using short, high-impedance transmission lines on either side of the overlap section. Approximating the transfer matrix for such a line for large Z_o and small θ gives

$$[T]\approx\begin{bmatrix}1 & jZ_o\sin\theta\\ 0 & 1\end{bmatrix}. \quad (26)$$

Equating this to the transfer matrix of a series inductor of value L in bandcenter we get

$$Z_o\sin\theta_c=\omega_c L. \quad (27)$$

As the transmission line is split into two sections, one either side of the overlap, then the reactance of each section need only be half of the total required value. The length of the inductive line may be determined using the identity

$$l=\frac{\theta_c u}{\omega_c}. \quad (28)$$

If we neglect the inductance of the overlap section, the overall resonant frequency will be too low. Analyzing the response of the three cascaded sections (transmission line, overlap, transmission line) enables us to obtain this resonant frequency, which can be adjusted to the correct value using an iterative method to shorten the lengths of the inductive transmission lines.

The combination of the shunt resonators and series elements gives a circuit configuration of the form of Fig. 14. Experiments have shown that the reference planes indicated in Fig. 15 give good results and hence were used for this design.

IV. Experimental Results

With the aid of a computer program to carry out the design procedure outlined, a fifth-degree filter, based on a 20-dB return loss prototype having a passband from 5 to 5.5 GHz, was constructed. A ground plane spacing of 0.070 in was used and the widths of the series sections were chosen to be 0.030 in, which is as small as is convenient for these elements. A grounded wall was placed between the central two resonators to reduce the effects of any waveguide modes propagating in the box.

Using small tuning screws over the overlap sections and the ends of the shunt resonators, slight adjustments in frequency of these elements are possible and the overall response of the device is shown in Fig. 16. These results

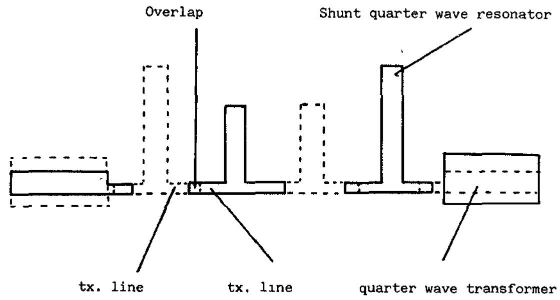

Fig. 14. Basic configuration of a degree five bandpass filter. Dotted lines indicate copper on the reverse side of the substrate.

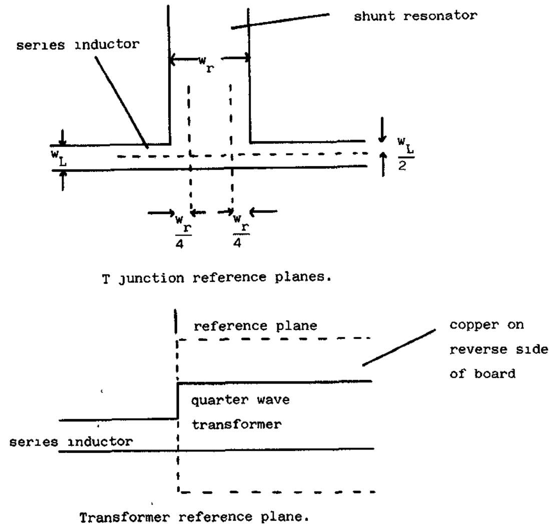

Fig. 15. Reference planes.

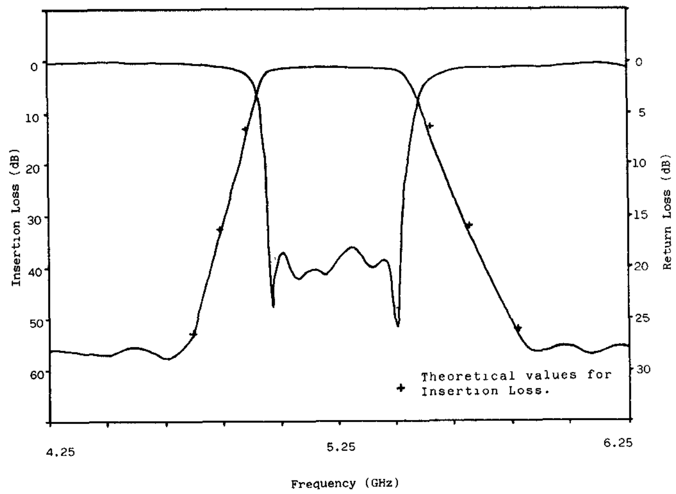

Fig. 16. Results for the prototype device.

compare almost exactly with the theoretical values produced by a computer analysis of the circuit, indicating the equivalent circuits used are a good approximation to the actual device.

We see that the selectivity on the low side of the passband is considerably better than on the high side. This is due to the nature of the inhomogeneous coupled lines, which make up the series elements of the filter. A further effect of these series sections is to make the stopband on the high side of the passband relatively narrow. The prototype filter held a rejection in excess of 60 dB (the designed stopband level) to a frequency of 7.8 GHz. This performance may be improved, however, by cascading the device with an integrated low-pass filter.

The insertion loss in midband was about 1 dB for an aluminum housing. This would be reduced if the housing were plated with copper or silver.

V. Conclusions

A design technique has been presented to realize a highly selective prototype in a true bandpass configuration. The relative simplicity of the design procedure enables narrowband suspended substrate stripline filters to be designed using short computer programs running on simple machines.

The results produced from the test device and the ease with which the designed performance was obtained, indicate the relative insensitivity of the filter response to both the shunt and series sections. This insensitivity means that manufacturing variations in the production of the device are not highly critical and, if tuning screws are used over the shunt resonators, reproducible results are possible.

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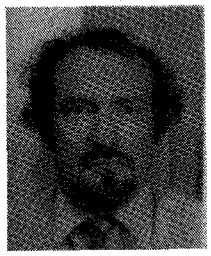

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Dr. Rhodes has been the recipient of five international research awards including the Browder J. Thompson, Guillimen–Cauer Awards, and the Microwave Prize. He has also been a member of several professional committees.